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ORO

OPERATIONS RESEARCH OFFICE
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STRATEGIC DIVISION

HUTSPIEL
a Theater War Game

by

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SUMMARY

PURPOSE

To develop, as part of the ORO program of war-gaming, a theater-level war game, employing the Goodyear Electronic Differential Analyzer (GEDA) installed at ORO, and directed to the study of the effects on a defense of stabilized positions in Western Europe of various employments of tactical atomic weapons and conventional air support.

FACTS

The game, called HUTSPIEL, was developed in the spring of 1955. It was played by two persons and represented a defense by BLUE of a fortified zone against RED attack, a situation such as might have occurred along the Rhine River in the summer of that year. To make best use of the analog computer (GEDA) the game was designed as deterministic (that is, any fixed decision by both sides would always produce the same results) and epochal (relying on a series of player-decisions made as each play of the game proceeded). One second of computer operation represented one day of combat. Between each computation, time was allowed for the players to study and record the interactions shown on the control board and to register thereon their next decisions. Play continued until a specified end point was reached.

The nature of the GEDA equipment required that most components of the game model be designed in terms of linear differential equations which could be solved in the machine by amplifiers with resistors or capacitors in feedback circuits. The non-linear operations of multiplication and division of one variable by another were performed by the Goodyear electronic multiplier and servo-multiplier units. A Mid-Century electronic function-generator was also used.

DISCUSSION

In the HUTSPIEL model the BLUE and RED theaters were depicted as symmetrical as to components but differing in input values and exchange ratios. There were five interacting target-weapon elements: ground forces (theater reserves and two combat sectors), atomic weapons, aircraft (in flight and on airfields), Class III and V supplies, and rail transport (rolling stock and network).

Supplies and troops could be transported between theater and sectors, and re-supply and reinforcement from the ZI were provided.

BLUE input values reflected the 1955 NATO troops and installations in France, Belgium, and West Germany; RED input values, the 1955 Soviet situation west of the Oder-Neisse river line. Initial conditions represented resources estimated to be available in event of an unexpected Soviet attack in the summer of 1955; full scale values, resources estimated to be available at the end of three months of war

The model also included mathematical representation of the concept of "latency" or performance degradation, applied to troops, mechanical equipment, and rail network. In respect to troops it symbolized the loss in combat effectiveness from strength losses, fatigue, and disorganization by adding to the usual types of casualties a relative number of "latents," both wounded and latents being designated as "ineffectives " Mechanical equipment (aircraft and locomotives) and rail network entered a latent or deadlined state in proportion to utilization and the amount of enemy-inflicted damage received. Recovery of troops, equipment, and rail network followed exponential time functions

Because of computer-equipment limitations, close support aircraft were aggregated as a single type plane for all missions The direct casualty rate from conventional air attack on troops was believed to be so low as to be insignificant in relation to other model components The disrupting effect of threatened damage from close-support sorties was, however, regarded as important and was depicted by increasing the number of ineffectives from atomic and ground attack in proportion to the number of supporting sorties flown against the troop target.

Two end points were used in the game, that occurring first terminating the play: (1) when the ratio of ineffective to active troops in a sector reached an arbitrary value, (2) when active troop strengths in a sector were so reduced as to be unable to man their line

Because of computer limitations or relative lack of significance, certain elements were omitted or but partially or indirectly represented. Truck transport on highway networks, effects of damage to airfield runways and maintenance facilities, shifts in the line of battle, the implications of battlefield mobility (e.g., proportion of armor to infantry), influences of weather and terrain, gathering and dissemination of intelligence

Specific discussion of game results is not included because these were dependent not only on the model but on input values which have by now lost most of their validity In HUTSPIEL the first steps essential to the development of a computer game were carried out: (1) education of the participants in construction and operation of an analog computer game, (2) study of the interactions of the theater model components as these were revealed by the computer; and (3) study of the effects of various employments of atomic weapons and conventional air support within the context of the input values and the model.

CONCLUSIONS

(1) The HUTSPIEL project demonstrated convincingly that the GEDA can be used successfully for deterministic war games in which player decisions are

a major element, its display board being a particularly desirable feature.

(2) The HUTSPIEL model contains many elements which individually and in combination deserve serious consideration for incorporation, unchanged or modified, in models of future theater war games.

PURPOSE

To develop, as part of the ORO program of war gaming, a theater-level war game, employing the Goodyear Electronic Differential Analyzer (GEDA) installed at ORO, and directed to the study of the effects on a defense of stabilized positions in Western Europe of various employments of tactical atomic weapons and conventional air support.

THE GAME

The game, called HUTSPIEL, was developed in the spring of 1955. It was a two-person game, one player making NATO decisions, the other representing the USSR. Because the addition of probabilistic variations would have more than doubled the computing equipment required, HUTSPIEL was designed as a deterministic game; that is, any fixed decision by both sides would always produce the same results. Although such a game is further from reality than a fully stochastic system, it provides a useful interim stage in the development of more general or more realistic models.

The participants began each play with the same assumed initial conditions, and made a series of decisions, selecting inputs from a range of values, until one of the agreed end points were reached. Most players initially had a bias toward some particular combination of strategies. As the machine and model were designed, there were actually only a few combinations which led to the best payoffs. Through repeated plays by various players these best strategies were discovered.

The framework of the game is the model, by which the designers of the game sought to represent the major components entering into and influencing the outcome of the game. The model is essentially a set of constructs and their reactions. Obviously a theater game, adapted to a machine which could be housed in one room, and playable in one morning by two people, cannot include in detail all the major elements which interact in an actual war. Some elements, believed to be least significant, were omitted entirely; others had to be aggregated, or averaged, or represented symbolically by depicting effects rather than their causative factors, or introduced as functions of other parameters. If the outcome of a game proves to be highly sensitive to variations in a component, however, this element must be depicted in detail and with high accuracy. Since sensitivity was often difficult to determine in advance and judgments varied widely, some components were represented in detail, which proved in play to have only marginal influence on the outcome; others had to be expanded or refined. The computer, the model, and the input values are discussed in the following sections.

THE COMPUTER

The nature of the HUTSPIEL game was determined by the analog computer on which it was played. War games played on digital computers usually consist of predetermined strategies, prepared before play begins and including instructions to cover all situations. This approach is dictated because of the time required to code instructions for the machine and to read out results to the players The analog computer (GEDA), on the contrary, is inherently good for a game relying on a series of player-decisions made as the play proceeds, a type of game referred to hereafter as "epochal." All elements of the HUTSPIEL model were displayed on the GEDA control panels, some 40 variables for each side. Each player, after registering his decisions by adjusting the appropriate dials and switches, pressed a "ready-button." The computer then operated for one second, representing one day of combat, or an "epoch." The resultant interactions from allocations and enemy attack were registered on meters from which the players could readily gather the information that would influence their decisions concerning the next day's operations Since the computer operated only after both ready-buttons had been pressed, the players could take time between computation periods to deliberate as long as they wished and to record meter readings as the play progressed. The continuous display feature is a valuable adjunct in war gaming, for, as players unfamiliar with HUTSPIEL demonstrated, an understanding of and feeling for the model and the game could be acquired in an hour or two spent at the control panels.

The ORO GEDA consisted of three linear and two non-linear consoles. In addition, an electronic function-generator manufactured by the Mid-Century Instrumatic Corporation was used. Each linear analog computing unit contained 24 amplifiers, making, with 8 auxiliary amplifiers, a total of 80. These computing amplifiers are the basic element of the electronic differential analyzer. With a resistor in the feedback circuit they are used for addition, subtraction, and sign-changing operations and multiplication of variables by constant coefficients. With a capacitor in the feedback circuit, amplifiers constitute an integrator which can sum a number of inputs and integrate them simultaneously One-half the available amplifiers were used as integrators, the others as summers, sign and scale changers, and in division circuits

The non-linear operations of multiplication and division of one variable by another were performed by Goodyear electronic multiplier and servo-multiplier units The Mid-Century electronic function-generator, an electronic curve-follower capable of generating any arbitrary single-valued function of a variable, was used in the interdiction component of the model.

The analog computor is especially adapted to solving problems expressed as sets of linear ordinary differential equations Since the number of non-linear computing elements was restricted, it was necessary to formulate the model so far as possible in terms of linear differential equations Representation of weapons effects and interactions in the model, therefore, often required a compromise between what was believed to approach reality and what could be feasibly incorporated in the electronic circuitry A subsequent section describes the mathematical construction of the model, and Appendix A details the equations by which the model components and their interactions were depicted in the machine

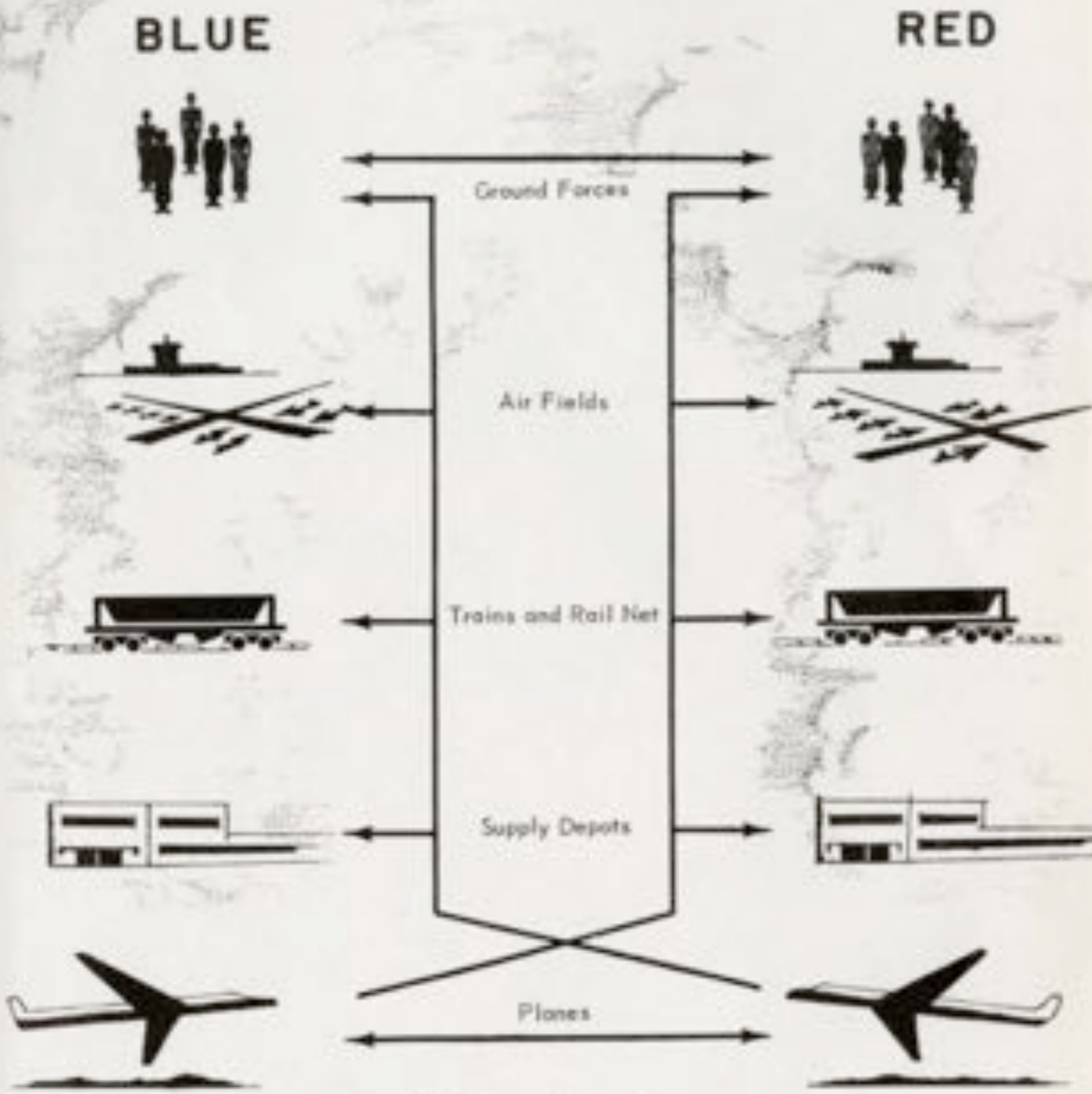

Fig. 1—Target–Weapon Components

Planes Attack Each Other and All Targets
Ground Forces Attack Each Other
Atomic Attacks All Targets Except Planes in Air

was conceived as symbolic of the condition of the whole troop unit and the numbers of latents were made dependent on the numbers of actual casualties, both wounded and latent appearing in the model as "ineffectives." Recovery from ineffectiveness was considered to follow an exponential time function and a fixed fraction of the total troops in the ineffective category was each day returned to the active category. (See Fig. 2.)

Mechanical equipment—aircraft and locomotives*—entered the latent or deadlined state in proportion to its utilization and the amount of damage from attack it had received. Recovery, as in the troop model component, was represented as an exponential time function.

The same approach was used in setting up repair rates for damage by interdiction to the railnet through-put capacity. In this case two levels of damage were assumed—moderate and severe. Repair functions for each were of the same form but different fractions of the remaining damage were repaired per unit of time. (See Fig. 3.)

In the treatment of close-air-support† in the model, aircraft, because of computer-equipment and game-time restrictions, were aggregated into a single type used for all missions: atomic weapons delivery, interception, conventional bombing, and ground-support operations. Indirect recognition was given, however, to the numbers of various plane types available to each side by setting limits to the number of sorties/day against various types of targets which would in reality be attacked by different kinds of aircraft. For example, a maximum of 2000 BLUE and 4000 RED close-support sorties/sector troops/day were permitted.

Casualties from conventional air attack on troops were not included in the model. Data from WWII and the Korean conflict indicated a casualty rate per close-support sortie so low that its significance would be lost in low-magnitude machine errors (noise level). It was believed, however, that the principal effect of close-support operations is the threat of damage, manifested in traffic disruptions, movement delays, neutralization and disorganization of troops—all of which essentially reduce the effectiveness of the enemy—and that this could be meaningfully expressed in terms of increased enemy troop casualties, especially in view of the "ineffective" category described above. Accordingly, both atomic and conventional casualties in a troop target were increased in proportion to the number of conventional sorties flown against the target by the opposing side. At a level of 2000 BLUE sorties/sector/day, for example, RED troop casualties in the sector were increased by a factor of two.

Two end points were used in the game, that occurring first terminating the play: (1) When the ratio of ineffective troops to effective (active) troops in a sector reached the arbitrary value of 0.33, it was considered that these divisions must withdraw if on the defensive, or if attacking must give up the assault; (2) When active troop strengths in a sector were so reduced that the remaining units could no longer be expected to hold their line.

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*Because locomotives were the rolling-stock element in short supply, damage and repair factors applying to them were used to represent the trains they drew.

†SAC was not played, most of its targets being beyond the limits of a theater.

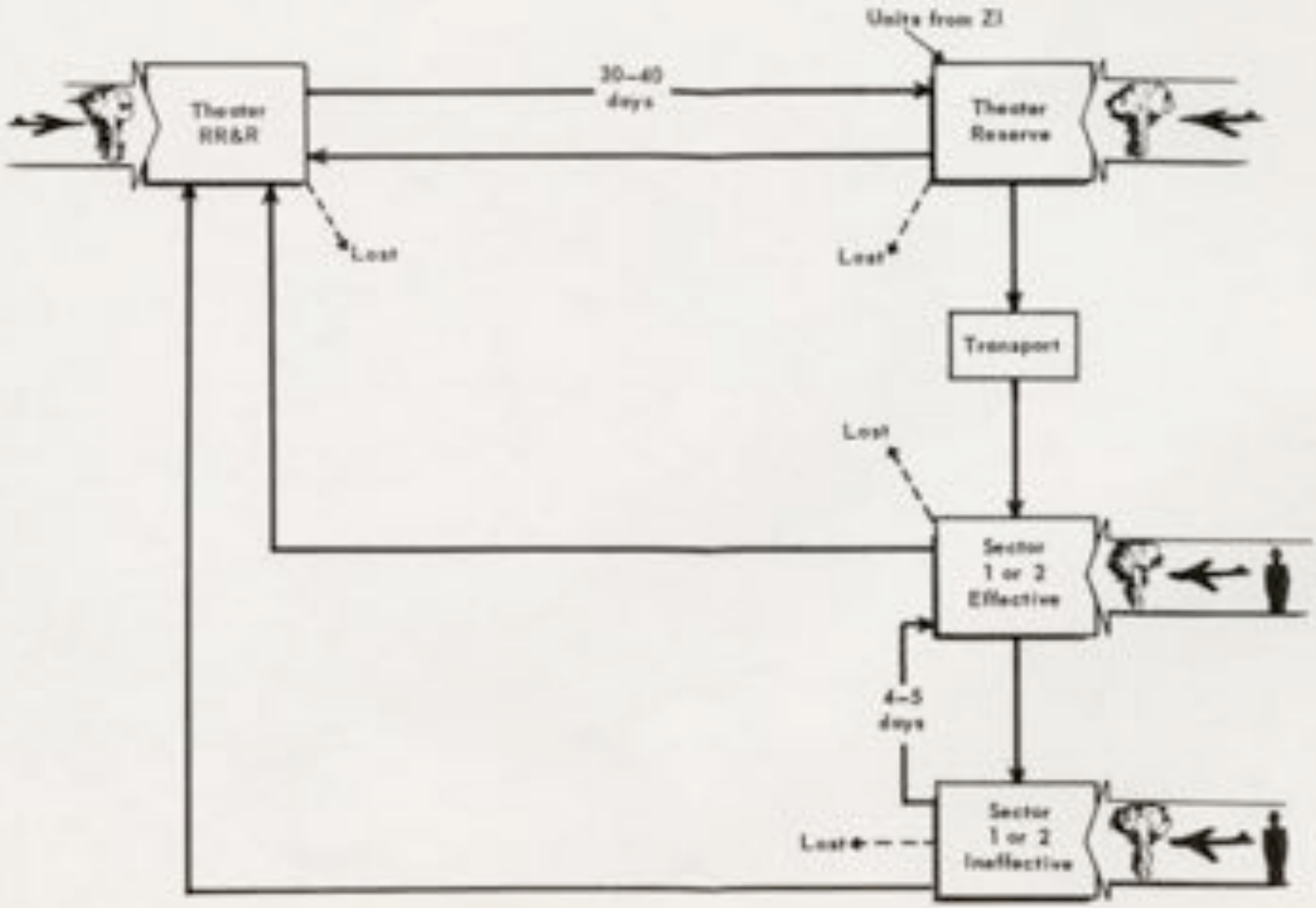


Fig. 2—Troop Component

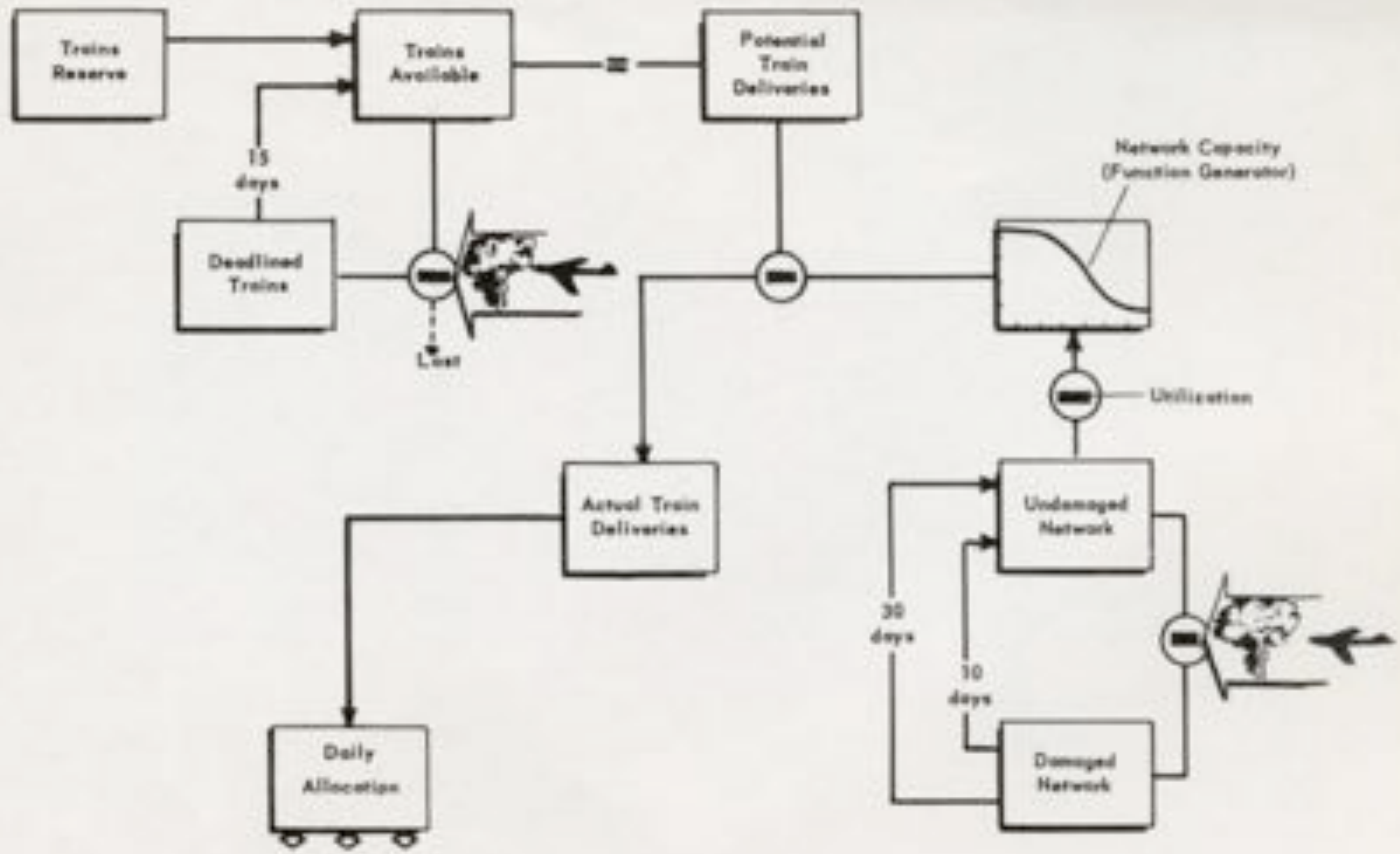


Fig. 3—Transport and Interdiction Component

In the majority of plays, both sectors on a side were played symmetrically; that is, all input variables were kept as closely in balance as possible The requirement to do so was imposed, particularly on BLUE which had smaller ground forces, by the necessity to avoid building up one sector at the risk of reaching an end point in the other

Because of computer limitations or lack of significance in relation to other components in the model, certain elements were omitted. Movement, except as this was involved in transporting supplies and troops by rail, was not represented in the game Truck transport and highway networks were not specifically included, because, apart from computer limitations, it was believed that the major portion of theater transport in Europe would be by rail and that the effects of interdiction on the small number of main raillines through destruction of key bridges would be a more significant factor than interdiction of highways and inland waterways The interdiction-damage component was so designed, however, that a fixed fraction of through-put capacity was invulnerable, which may be considered to represent the capability of transport by truck. Shifts in the line of battle, resulting from attack and withdrawal, were not depicted. Relationships between movement and casualty rates were not introduced beyond recognition of differing casualty rates appropriate to attack and defense; nor were the implications of battlefield mobility (e.g., proportion of armor to infantry) included. Also omitted were the influences of weather and terrain, intelligence gathering and dissemination, and, less significant, the effects of damage to airfield runways and aircraft maintenance facilities.

MATHEMATICAL CONSTRUCTION OF THE MODEL*

As has been pointed out, the nature of the GEDA equipment necessitates that most components of the model be designed in terms of linear differential equations The simplest linear form is one of constant return for a certain level of effort against a given target type; for example, a fixed amount of damage is assumed for each atomic weapon regardless of the target inventory This is satisfactory only as a first approximation, because the destruction per weapon usually depends on the size or the number of targets in the system. A better form would relate the damage per unit attack to the target inventory remaining. If the damage is made directly proportional to the target inventory at the time of attack, the effect of attacks on a target system may be represented by a set of linear differential equations which may be written as:

*See Appendix A for detailed equations by which model components and their interactions were depicted.

$$dx_i = \Sigma \alpha_{ij} x_i dE_{ij} \qquad (1)$$

where dx_i = units of i^{th} target destroyed
α_{ij} = fraction of target destroyed per unit attack effort
dE_{ij} = units of attack effort of j^{th} type allocated to i^{th} target.

Any other relationship for damage to targets will result in differential equations of a non-linear form and will considerably increase the amount of computing equipment needed. The analog of each term of Eq. 1 is simply an integrator with feedback, the quantity x_i being represented by the integrator and the amount of feedback controlled by the scale of attack (E_{ij}).

As long as the change in x during any interval is not large the equations for the epochal* changes may be written as

$$\Delta x_i = \alpha x_i \Delta E_{ij} \qquad (2)$$

where Δx_i = change in x_i during epoch (in terms of aircraft, days of supply, etc.)
ΔE_{ij} = attack effort during epoch (in terms of sorties, A-bombs, etc.).

The terms in Eqs A26 and A27 (Appendix A) that relate to attack by sorties and atomic weapons are of this form.

The mathematical statement of the model in Appendix A is, in general, in terms of the epochal changes in the variables expressed in difference equations, such as Eq. 1 It should be pointed out that, while these approximately represent the model, the computer actually integrates, over each epoch, the differential equations, such as Eq. 2, which are equivalent to the difference equations

The equations in the troop model for the conventional casualties resulting from the interactions of ground forces (see Eq. A8a and A8b) appear to be of the same form as Eq. 2 In simplified form they may be written as

$$\Delta N = \alpha N \bar{N} \bar{U}, \qquad (3)$$

where α = numerical coefficient
ΔN = change in friendly troops during an epoch,
N = number of friendly troops,
$\bar{N}$ = number of enemy troops,
$\bar{U}$ = utilization of enemy troops.

* "Epoch" = one-second computator operation equivalent to one day of combat in play

But, whereas the quantity ΔE in Eq. 2 is fixed for each epoch by player decision, the quantity, $\dot{N}U$, varies during the computation interval. The relation, therefore, is non-linear, and the analog of equations of the form of Eq. 3 requires multiplier units in the feedback circuits of the integrator representing the quantities N

Several target systems in the game were found to have characteristics which could be reasonably approximated by a relationship of the form of Eq. 1. Examples of these follow

Attack on Supplies   Supplies are distributed in a theater in a large number of depots of varying size, the larger being at Army COMZONE level and few in number, the smaller and much more numerous being forward with Corps and Divisions   Data on the US Seventh Army supply system and somewhat sketchy information on Soviet stores in East Germany showed that, if the depots were arranged in order from large to small by rank, lettering 1 equal the largest, 2 the next largest, etc , the size of the depots (in terms of days of supply or tons in the depot) could be approximated by a power series. This may be written as

$$A_n = A_1 a^{n-1} \tag{4}$$

where A_1 = size of largest depot (in days of supply),
A_n = size of n^{th} depot (in days of supply),
a = numerical constant (between .85 and .95),
n = rank order of depot (1 equals largest, etc.).

The total quantity A_t stored in a system of N depots is, therefore,

$$A_t = A_1 \sum_{n=1}^{N} a^{n-1}$$
$$= A_1 \left[\sum_{n=1}^{\infty} a^{n-1} - \sum_{n=N+1}^{\infty} a^{n-1} \right]$$
$$= A_1 \left[\sum_{i=0}^{\infty} a^i - a^N \sum_{i=0}^{\infty} a^i \right]$$
$$= A_1 \left[(1-a^N) \sum_{i=0}^{\infty} a^i \right]$$
$$= A_1 \left[1-a^N / 1-a \ . \right]$$

If N is large, the term $(1-a^N)$ can be neglected and

$$A_t = A_1 / 1-a \tag{5}$$

The error introduced by dropping the term A_{N+1} in the case of the finite systems being considered is quite small as long as the number of depots in the system is large   The values of "a" for the various depot systems under study (Class III and V, forward and theater) ranged between 75 and .95 and the number of depots in each was usually greater than 25. Thus the total quantity stored in the system can be satisfactorily approximated by Eq. 5.

In certain of the target systems only one atomic weapon is required to destroy each depot. For this case and with a rank order distribution as above, the effects of attack are as follows

If the total quantity stored is $A_1/(1-a)$, and if the largest depot, A_1, is attacked first, then the fraction of the total system destroyed by the first attack is $(1-a)$, and the quantity remaining in the system is $A_1a/(1-a)$ Now, since the size of the second depot in the system is A_1a, the fraction of the quantity remaining in the system destroyed by the second bomb is $(1-a)$, which is the same fraction as was destroyed by the first bomb. Thus, the fraction $(1-a)$ of the remaining stock is destroyed by any bomb, and the analog of Eq. 1 represents the effects of attack on a target system of the type of Eq. 2 if σ in Eq. 1 is set equal to $(1-a)$

An example of a target system meeting the above conditions is found in BLUE's packaged Class III storage system where the depots are all small enough that 80KT weapons or less can average 90 percent or higher destruction at each depot.

It is perhaps trivial to point out that depot systems containing depots too large to be destroyed by a single weapon can be treated in a similar fashion as long as the amount destroyed per weapon can be approximated by an expression of the form of Eq. 2 Eqs A26 and A27 contain the attack terms based on the systems as described above

In these two cases sufficient information is presumed available to each side to permit planning to attack the largest depots first.

Attack on Airfields Aircraft on airfields are an example of another type of target system that can be approached in the same manner In this case the number of aiming points (airfields) is assumed to remain constant, while the number of aircraft varies The average number of aircraft on a field is, therefore, proportional to the total number in the system.

The assumption that squadrons can be shifted from one base to another and that those fields which suffer atomic attack retain their usefulness as operational bases, coupled with imperfect intelligence, leads one to a model for attacks on airfields in which the number of aircraft destroyed per atomic weapons is proportional to the number remaining in the system Again the analog of Eq. 1 is adopted. This is represented by the third and fourth terms of Eq. A22 and the third term of Eq. A23.

Attack on Troops A target system made up of mobile and diffuse elements wherein casualties occur among ground troops from atomic weapons is the third type of system for which the feedback model is useful as an analog. Considering that atomic weapons are primarily used for area effects, then the casualties produced are assumed proportional to the rate of weapons employment and the density of troops. It is also to be noted that weapons fired at targets on which intelligence is imperfect will produce casualties in proportion to the number of troops in the target area. For this problem the length of the line of contact does not change and the number of troops in the beaten zone is proportional to the number of troops in the sector

The net result of the foregoing, is that, to a first approximation, the casualty-producing effectiveness of atomic weapons is evaluated at a given level of forces, and for other levels the return per round is assumed to scale linearly with the number of divisions in the sector being fired upon. Eqs A7a, A7b and the third term of Eq. A1 represent interactions of this form.

INPUT VALUES

The term, "input values," covers all the numerical values assigned to the elements of the model, including those indicated by the equations in Appendix A, those implicit in the damage and repair functions, and the initial conditions and full scale values assigned to model components The model, of course, permits use of a variety of input values, although minor changes in the computer wiring might in some cases be required to accomodate large variations

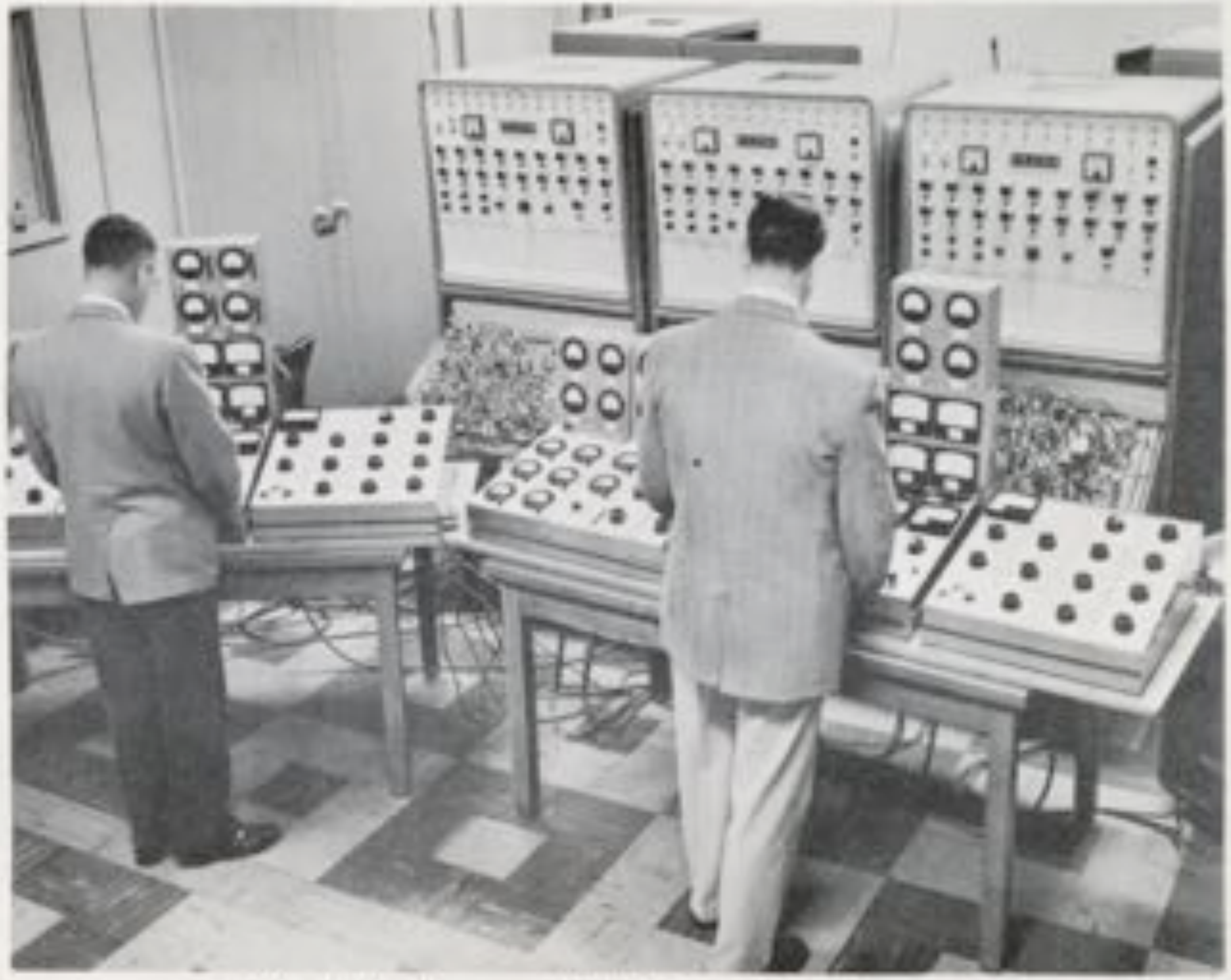
No detailed discussion of the input values used in HUTSPIEL is here included, in many cases whatever validity could be claimed for them has vanished in the changing situation during the years since 1955. Nor is a discussion of the detailed results of plays included. At this stage in the development of war gaming, it should not be necessary to labor the point that specific numerical results from a game are meaningful to a real situation only to the degree that the model and the input values depict the real situation, and even if the model itself be a fair approximation of reality, the validity of the results is degraded if the inputs range around values which are far from the real means in the 1958 situation.

It will be evident from a glance at the components of the model that some types of data could be easily collected with considerable confidence in their approximation to reality; for example, the number and contents of US Class III and V depots in West Germany were known. Other types of data, such as areas of effectiveness of atomic weapons, at least had the sanction of general usage in military estimates For some inputs, such as the relative proportion of "latent" to actual casualties, little or no data existed and arbitrary values had to be assigned. In connection with other values there was disagreement among experts for example, in regard to the interdiction component—what rate or rates should be allowed for recovery of through-put capacity after atomic attacks on key railway bridges? How fast and how completely could conversion to truck transport be accomplished?

To some inputs the model showed great sensitivity As long as a steady resupply of Class III and V at one day of supply/division/day from the BLUE and RED ZIs was maintained, attacks on theater depots had very little effect on the outcome of play. Without this resupply, attacks on theater depots were extremely profitable. But no data were available by which to determine how these ZI supply inputs should be varied, and it was outside the provenance and resources of HUTSPIEL to engender data by including these components in the model. Here was a boundary area in which other games designed to explore the effects of SAC interdiction and of enemy attacks on the US sea supply lines, ports, and beaches might contribute materially to a more meaningful theater game.

PLAY OF HUTSPIEL

Play of HUTSPIEL was not usually conducted as a formal contest in which each side sought only to defeat his opponent. Instead, both players cooperated by agreeing in advance of each play on the main strategies to be followed and by exchange of information and discussion as the play progressed. By prior agreement, for example, BLUE might decide to expend his atomic weapons on RED ground troops and his conventional air attacks on RED's transport network, while RED concentrated his atomic and conventional air attack on BLUE airfields



HUTSPIEL Players at GEDA Control Panels

information on the development of these strategies would be freely exchanged throughout the play so that if, for instance, it became evident that the BLUE air force had been reduced to a minor threat, RED could husband his weapons until repair and resupply had again built up these targets By a series of such plays it was possible to construct a matrix of strategies and to give an indication of combinations which, insofar as the model could indicate, might enable BLUE to hold out longest against RED attack.

This approach was in the main dictated by the object of the exercise, to study the effects of varying employments of atomic weapons and conventional air support within the context of the game It was also imposed by the absence of an "intelligence" component within the model because of limited computer equipment; for, while this essential feature of a formal war game could have been supplied (and sometimes was) by methods extraneous to the machine, the chief concern of HUTSPIEL was the development and testing of the model and its interactions on the GEDA itself. Thus, the first steps essential to the development of a computer game were carried out: (1) education of the participants in construction and operation of an analog computer game, (2) study of the interactions of the theater model components as these were revealed by the computer, and (3) study of the effects of various employments of atomic weapons and conventional air support within the context of the model.

CONCLUSIONS

(1) The HUTSPIEL project demonstrated convincingly that the GEDA can be used successfully for deterministic war games in which player decisions are a major element, its display board being a particularly desirable feature

(2) The HUTSPIEL model contains many elements which individually and in combination deserve serious consideration for incorporation, unchanged or modified, in models of future theater war games

Appendix A

DEFINITION OF SYMBOLS AND EQUATIONS USED IN HUTSPIEL

SYMBOLS

(a) Equations and symbols are symmetrical and are herein given for the BLUE side only The RED interaction symbols and equations may be obtained by interchanging the BLUE and RED symbols

(b) The bar over a symbol, as in $\bar{N}$, refers to the RED side A symbol without a bar refers to the BLUE side unless it is obvious that RED is intended, as in targets attacked by BLUE weapons

(c) The following symbols are used.

A	Class V supplies
C	casualties (cause defined by symbol following)
E	damage component
F	airplanes
I	interdiction factor
P	posture (attack or defense)
R	Class III supplies
S	aircraft sorties
T	transport (railway trains)
U	utilization (fraction of total capability employed in epoch)[1]
W	atomic weapons (damage computation is made in terms of optimum weapon size, allocation in terms of equivalent 5 KT weapons)

The lower case is used when the items appear as subscripts

(d) The following subscripts refer to areas

0; theater
1; sector 1
2; sector 2
i, any sector

Thus R_0 represents theater Class III; A_2 represents sector 2 Class V

The subscript 0 outside a parenthesis denotes input rates into theater; e.g , $(N)_0$ is the daily input of divisions into the theater

(e) A multiple subscript shows specific allocation for a given day of combat (epoch), identifying the item, its location, and its end use. For example, T_{a01} indicates trains allocated to move Class V supplies from theater to sector 1, S_{a1}, the BLUE aircraft sorties allocated to attack RED Class V depots in sector 1, the bar being omitted over the subscript since it is obvious that BLUE sorties are directed against RED.

(f) Whenever a quantity exists in both effective and ineffective (or deadlined) states, the superscript (*) indicates the effective or combat ready state and the superscript dagger (†) indicates the ineffective or deadlined state

(g) The Greek letters α, β, γ, λ are constants, related to but not the same as the exchange ratios They are used as follows:

- α when the casualty effect is proportional to the product of friendly and enemy forces
- β when the casualty effect is proportional to enemy forces only
- γ when the casualty effect is proportional to friendly forces only
- λ when the recovery effect is proportional to friendly forces only

When it is necessary to identify the constant, symbols are placed in parentheses following the Greek letter, e.g., $\alpha(P_1, \bar{P}_1)$

- τ represents capacity of trains τ_n divisions per train, τ_r division days of Class III per train.
- δ represents the ratio of the numbers of troops rendered ineffective to the casualties sustained by effective troops The value is different for troops in attack than for those in defense
- Δ used before another symbol to indicate the daily (epochal) change in that quantity, e.g., ΔN_1 represents the daily change in the total troops (divisions) in sector 1

INTERACTION EQUATIONS

TROOPS

I Theater Troops Combat Ready Reserve:

$$\Delta N = (\Delta N)_0 + \lambda_0(\Delta N^{\dagger}) - \gamma(\bar{\bar{w}}, N)N\bar{\bar{w}}_n - \gamma(\bar{\bar{S}}, n)N\bar{\bar{S}}_n - \Sigma(\Delta N_i)_t. \quad \text{(A1)}$$

After the equality the meaning of the terms is as follows

- 1st term. Arrivals from ZI, a function of time
- 2nd term. Reactivation of ineffective theater troops
- 3rd term. Losses caused by enemy atomic weapons
- 4th term Losses caused by enemy conventional aircraft sorties (see text)
- 5th term. Troops transported out of reserves to sectors

Transfers from Theater to Sectors:

$$(\Delta N_i)_t = \tau_n t_{noi} + \text{intersector transfer cancelling out.} \quad \text{(A1.a)}$$

(see also Eq. A10)

II Theater Troops in RR&R.

$$\Delta N^{\dagger} = \tau_w \sum_i (\Delta N_i)_{cw} - \lambda_0 N^{\dagger} - \gamma(w, n)N^{\dagger}\bar{\bar{w}}_n - \gamma(\bar{\bar{S}}, n)N^{\dagger}\bar{\bar{S}}_n \quad \text{(A2)}$$

After the equality the meaning of the terms is as follows

- 1st: disorganized troops surviving atomic attack in sectors
- 2nd: reactivation of ineffective troops
- 3rd. losses caused by enemy atomic weapons
- 4th. losses caused by enemy conventional aircraft sorties

III Troops in Sector i.

$$N_i = N_i^* + N_i^\dagger . \tag{A3}$$

where i = 1 or 2 Total troops in sector are the sum of the effective and ineffective troops

Total Change in Troops in Sector i.

$$\Delta N_i = \Delta N_i^* + \Delta N_i^\dagger . \tag{A4}$$

Total Change in Effective Troops in Sector i.

$$\Delta N_i^* = -(\Delta N_i^*)_{c\bar{\bar{w}}} - [1 + \delta(P)]\,[(\Delta N_i^*)_{c\bar{\bar{g}}} + (\Delta N_i^\dagger)_{c\bar{\bar{g}}}] + \lambda_m N_i^\dagger + (\Delta N_i)_t . \tag{A5}$$

The meaning of the terms on the right is as follows

- 1st: troops removed from effective state as a result of enemy use of atomic weapons, including those moved to theater RR&R for reorganization.
- 2nd. losses due to conventional ground and close-support operations plus those rendered ineffective by these losses The direct casualties produced by close-support aircraft operation were omitted as insignificant (see A9)
- 3rd. ineffective troops recovering to effective state during epoch.
- 4th. net arrivals transported into sector

Following Eq. A3 it is obvious that:

$$(\Delta N_i)_{c\bar{\bar{w}}} = (\Delta N_i^*)_{c\bar{\bar{w}}} + (\Delta N_i^\dagger)_{c\bar{\bar{w}}} \tag{A3.a}$$

$$(\Delta N_i)_{c\bar{\bar{g}}} = (\Delta N_i^*)_{c\bar{\bar{g}}} + (\Delta N_i^\dagger)_{c\bar{\bar{g}}} \tag{A3.b}$$

$$(\Delta N_i)_{c\bar{\bar{s}}} = (\Delta N_i^*)_{c\bar{\bar{s}}} + (\Delta N_i^\dagger)_{c\bar{\bar{s}}} \quad \text{(direct casualties from air attack, neglected)} \tag{A3.c}$$

Similar to Eq. A5, one has

Total Change in Ineffective Troops in Sector i.

$$\Delta N_i^{\dagger} = -(\Delta N_i^{\dagger})_{c\bar{w}} - (\Delta N_i^{\dagger})_{c\bar{n}} - (\Delta N_i^{\S})_{c\bar{s}}$$

$$- \lambda_n N_i^{\dagger} + \delta(P)(\Delta N_i^{*})_{c\bar{n}}$$

$$- \delta(w_i)(\Delta N_i^{\S})_{c\bar{w}} . \qquad \text{(A6)}$$

where the terms on the right are as follows

1st. troops removed from the ineffective state as a result of enemy use of atomic weapons (in close support), including casualties and those transferred to theater RR&R for reorganization and recuperation.
2nd. casualties produced by conventional enemy ground action against ineffective component.
3rd. direct casualties produced by enemy close-support aircraft sorties against ineffective component omitted, see A9
4th. ineffective troops rendered effective during epoch.
5th. the production of ineffective troops from the effective troops through conventional battle casualties
6th. the production of ineffective from effective troops through atomic battle casualties These are survivors from units in fringe areas—not sufficiently depleted to warrant transfer to theater RR&R.

The atomic losses in the sector computed by

$$(\Delta N_i^{*})_{cw} = \gamma(\bar{W}_i, n) N_i^{*} \bar{W}_{ni}\left[1 + \frac{\bar{S}_{ni}}{\bar{S}_0}\right] \text{ and} \qquad \text{(A7.a)}$$

$$(\Delta N_i^{\S})_{cw} = \gamma(\bar{W}_i, n) N_i^{\dagger} \bar{W}_{ni}\left[1 + \frac{\bar{S}_{ni}}{\bar{S}_0}\right] \qquad \text{(A7.b)}$$

include both the actual casualties and the "organizational casualties," i.e , those organizations so disrupted that complete reorganization in theater RR&R is necessary

The sector casualties are computed by

$$(\Delta N_i^{*})_{c\bar{n}} = \alpha(P_i, \bar{P}_i) N_i^{*} \bar{N}_i^{*} \bar{U}_i \left[1 + \frac{\bar{S}_{ni}}{\bar{S}_0}\right] \qquad \text{(A8.a)}$$

$$(\Delta N_i^{\dagger})_{c\bar{n}} = \alpha(P_i, \bar{P}_i) N_i^{\dagger} \bar{N}_i^{*} \bar{U}_i \left[1 + \frac{\bar{S}_{ni}}{\bar{S}_0}\right]. \qquad \text{(A8.b)}$$

where the losses are determined by the postures adopted by each side and the α's, and are proportional to the density of friendly troops, as well as the numbers of <u>effective</u> enemy troops multiplied by the utilization of enemy troops, and by the close-air-support effect (see A9) $\bar{S}_0$ is a constant.

The direct casualties produced by close-air-support attacks were calculated by

$$(\Delta N_i^{*,\dagger})_{cs} = \beta \bar{S}_{ni} \frac{N_i^{*,\dagger}}{N_i} \qquad \text{(A9)}$$

and were determined to be insignificant with respect to the other losses

The change in the i^{th} sector troops brought about by transportation transfer from the theater and to and from the j^{th} sector is given by

$$(\Delta N_i)_t = \zeta_n [t_{noi} + t_{nji} - t_{nij}]. \qquad \text{(A10)}$$

TRANSPORTATION

The total number of trains in the theater, T, is

$$T = T_{reserve} + T^{\dagger} + T^{*} \qquad \text{(A11)}$$

Reserve trains are civilian equipment which can be requisitioned for military use. The change in the number of serviceable trains, T^*, is given by

$$\Delta T^* = -\xi U_t T^* + \lambda_t T^{\dagger} - \beta_{ts} \bar{S}_t - \beta_{tw} \bar{w}_t + T_M . \qquad \text{(A12)}$$

where the terms on the right are as follows:

1st: rate of deadlining depending on utilization of trains, the constant of proportionality being ξ,
2nd. rate of return to serviceability from deadlined trains,
3rd. losses caused by enemy conventional air sorties,
4th. losses caused by enemy atomic weapons on marshalling yards,
5th. trains requisitioned from the civilian reserve

The change in the deadlined trains is given by

$$\Delta T^{\dagger} = \xi U T^* + \eta_{st} \beta_{ts} \bar{S}_t + \eta_{wt} \beta_{tw} \bar{w}_t - \lambda_t T^{\dagger} \qquad \text{(A13)}$$

Here the terms on the right originate as follows.

1st: rate of deadlining from usage;
2nd. trains damaged by enemy conventional aircraft, which can be made serviceable, η being the fraction of such trains,
3rd. trains damaged by atomic weapons, which can be made serviceable,
4th. trains made serviceable and removed from deadlined status

The total trains dispatched per day, $t^{\dagger}$, is given by

$$t^{\dagger} = \frac{UT^{*}}{D}, \tag{A14}$$

where D is the turn-around time in days The actual total train arrivals per day is then given by

$$t = t^{\dagger} I, \tag{A15}$$

$t^{\dagger}$ being the potential trains dispatched.

The total train arrivals is then allocated among the various uses for transportation.

INTERDICTION

The interdiction factor I is determined by the stress, χ, on the transportation network.

$$I = f(\chi), \tag{A16}$$

i.e., I is a function of χ This is a monatomically decreasing functorial relationship, being at $I = 1$ for $\chi = 0$, and decreasing with a sharp bend in the curve with increasing stress The stress for which $I = 1/2$ is denoted $\chi_{½}$ The function (A16) is placed on the function generator The stress is the sum of a utilization and a damage component

$$\chi = \epsilon_u UT^{*} + \epsilon_e E \tag{A17}$$

The constants, ϵ, are determined by those stresses which reduce I to 1/2

$$\epsilon_u = \left[\frac{\chi}{UT^{*}}\right]_{I = 1/2}, \; E = 0; \tag{A18.a}$$

$$\epsilon_e = \left[\frac{\chi}{E}\right]_{I = 1/2}, \; U = 0 \tag{A18.b}$$

The damage E is initially zero; its change is given by

$$\Delta E = \beta_{1s}\bar{\bar{S}}_1 + \beta_{1w}\bar{\bar{w}}_1 - \lambda_{11}E_1 - \lambda_{12}E_2, \tag{A18}$$

where β_{1w} is taken to be unity and the damage measured in terms of that produced by 5 KT atomic weapons There are two kinds of damage—that, E_1 requiring 10 days $(1/\lambda_{11})$ to repair; that, E_2 requiring 30 days $(1/\lambda_{12})$ to repair

$$E = E_1 + E_2 \tag{A19}$$

The ratio of damage of type 1 to type 2 is r, then

$$\Delta E_1 = r\beta_{1s}\hat{S}_1 + r\beta_{1w}\bar{\bar{w}}_1 - \lambda_{11}E_1, \tag{A20.a}$$

$$\Delta E_2 = (1-r)\beta_{1s}\hat{S}_1 + (1-r)\beta_{1w}\bar{\bar{w}}_1 - \lambda_{12}E_2 \tag{A20.b}$$

AIRCRAFT

The total aircraft is the sum of those combat-ready and deadlined.

$$F = F^{*} + F^{\dagger} \tag{A21}$$

Changes in the numbers of combat ready aircraft are in accordance with the following equation.

$$\Delta F^{*} = -\beta_s \hat{S}_s - \gamma_s S - \alpha^{*}_{fs} F^{*} \hat{S}_f - \alpha^{*}_{fw} F^{*} \bar{\bar{w}}_f + \lambda_f F^{\dagger}, \tag{A22}$$

where the terms on the right have the following meaning:

1st. losses caused by enemy interception,
2nd. deadlining through utilization,
3rd. losses caused by enemy conventional air attack on airfields,
4th. losses caused by enemy atomic attack on airfields,
5th. deadlined aircraft made combat-ready

In the third and fourth terms the α's are different for the combat-ready and deadlined since part of the former will be in flight, and not subject to the attack.

Changes in the numbers of deadlined aircraft are governed in like fashion by:

$$\Delta F^{\dagger} = \gamma_f S + [\eta_{sf}\alpha^{*}_{fs}\hat{S}_f + \eta_{wf}\alpha^{*}_{fw}\bar{\bar{w}}_f] F^{*} - [\alpha^{\dagger}_{fs}\hat{S}_f + \alpha^{\dagger}_{fw}\bar{\bar{w}}_f] F^{\dagger} + (\Delta F)_0 - \lambda_f F^{\dagger}, \tag{A23}$$

where the terms on the right have the following meaning:

1st: deadlining through utilization,
2nd. aircraft which can be made serviceable from those damaged by enemy conventional air and atomic weapon attacks on airfields,
3rd. deadlined aircraft destroyed by enemy conventional air and atomic weapon attacks on airfields;
4th. resupply of aircraft from ZI;
5th. deadlined aircraft made combat ready

The daily sorties are given by:

$$S = U_f F^{*}, \tag{A24}$$

and are allocated among the various missions

$$S = \sum_j S_j \tag{A25}$$

SUPPLIES

Supplies arrive from the ZI, are transported, consumed, and destroyed by enemy conventional and atomic attacks

<u>Class III</u>

$$\Delta R = (\Delta R)_0 - \tau_r \sum_i t_{roi} - \gamma_{rso} R\bar{\bar{S}}_{ro} - \gamma_{rwo} R\bar{\bar{w}}_{ro}, \tag{A26}$$

where the constant τ_r is the division-days of POL supply carried per train.

The sector Class III supplies are consumed at a constant rate per troop unit.

$$\Delta R_1 = \tau_r t_{ro1} - N_1 - \gamma_{rs2} R_1 \bar{\bar{S}}_{r1} - \gamma_{rwi} R_1 \bar{\bar{w}}_{r1}, \tag{A27}$$

(and similarly for sector 2); the first term being that transported in, the second that consumed at the rate of one division-day per division, the third and fourth being losses due to enemy conventional and atomic attacks

Aircraft Class III is measured in sorties of supply and is consumed proportional to utilization.

$$\Delta R_f = \sigma_r \tau_r t_{rof} - S - \gamma_{rsf} R_f \bar{\bar{S}}_{rf} - \gamma_{rwf} R_f \bar{\bar{w}}_{rf}, \tag{A28}$$

where σ_r is the number of sorties of supply per division-day of supply

The transport Class III is solid fuel.

$$\Delta R_t = \tau_t t_{rot} - \sigma_t t \tag{A29}$$

The losses caused by enemy action were neglected. The unit of measurement is the kiloton of coal. Hence τ_t is the kilotons carried per train, and σ_t is the kilotons consumed per train delivery per day

<u>Class V Supplies</u>:

The equations representing Class V supplies are similar to those for Class III, with these exceptions transport of course does not consume Class V; the troops in sectors consume Class V in proportion to troop utilization.

$$\Delta A = (\Delta A)_0 - \tau_a \sum_i t_{aoi} - r_{aso} A\bar{\bar{S}}_{ao} - \gamma_{awo} A\bar{\bar{w}}_{ao}, \tag{A30}$$

$$\Delta A_i = \tau_a t_{aoi} - U_i N_i - \gamma_{asi} A_i \bar{S}_{ai} - \gamma_{awi} A_i \bar{w}_{ai}, \tag{A31}$$

where i = sector 1 or 2

$$\Delta A_f = \sigma_a \tau_a t_{aof} - S - \gamma_{asf} A_f \bar{S}_{af} - \gamma_{awf} A_f \bar{w}_{af} \tag{A32}$$

END POINT RATIO

The end point ratio (see text) is defined by:

$$B_i = \frac{N_i^{\dagger}}{N_i^{*}} \tag{A33}$$

and the end point has been reached whenever $B_i \geq B_p$, where B_p is a constant (= 0.33)